OLIMPIADA DE MATEMATICĂ-FAZA LOCALĂ
CLASA a VII a

BAREME ŞI SOLUȚII ORIENTATIVE

SUBIECTUL I

a) $\sqrt{\frac{1}{1\cdot 4}+\frac{1}{2\cdot 6}+\frac{1}{3\cdot 8}+...+\frac{1}{49\cdot 100}}=\sqrt{\frac{1}{2}\left(\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+...+\frac{1}{49\cdot 50}\right)}=$ 1p

$\sqrt{\frac{1}{2}\cdot\frac{49}{50}}=\sqrt{\frac{49}{100}}=\frac{7}{10}$ 2p

b) $ab-a-b+1>0$ 1p

$(a-1)(b-1)>0$ 1p

$a-1$ şi $b-1$ au acelaşi semn 1p

Finalizare, deoarece $a+b<2$ 1p

SUBIECTUL II

Notăm $|a_1-a_2|=|a_2-a_3|=...=|a_{2010}-a_{2011}|=|a_{2011}-a_1|=t,\ t\geq 0$ 1p

Rezultă $a_1=a_2\pm t,\ a_2=a_3\pm t,\ ...,\ a_{2011}=a_1\pm t$ 2p

Adunând egalitățile obținem $2011=2011+(\pm t\pm t\pm...\pm t)$ 1p

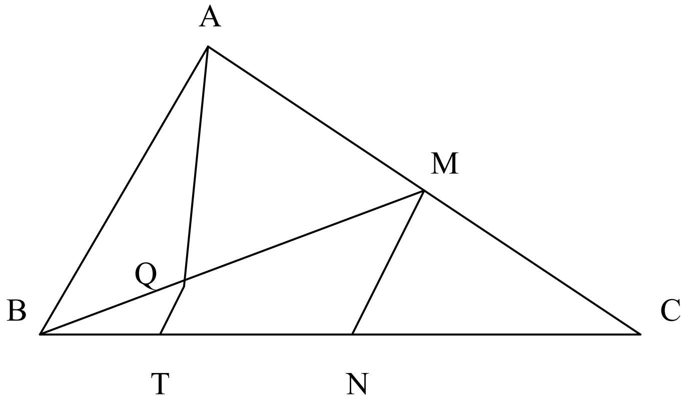
$0=2011\cdot(\pm t)$ 1p

$t=0$ 1p

Finalizare $a_1=a_2=...=a_{2011}=1$ 1p

SUBIECTUL III

Figura 1p



Construim $MN\parallel AB\Rightarrow N$ mijlocul lui $[BC]$ rezultă $A_{BMN}=\frac{1}{2}A_{BMC}=\frac{1}{4}A_{ABC}$ 1p

Notăm $\frac{BQ}{BM}=k$

Deoarece $QT\parallel MN\Rightarrow\Delta BQT\approx\Delta BMN\Rightarrow A_{BQT}=k^2A_{BMN}$ 1p

Deci $A_{BQT}=\frac{k^2}{4}A_{ABC}$ 1p

$A_{ABQ}=\frac{BQ\cdot d(A,BM)}{2}=\frac{k\cdot BM\cdot d(A,BM)}{2}=k\cdot A_{ABM}$

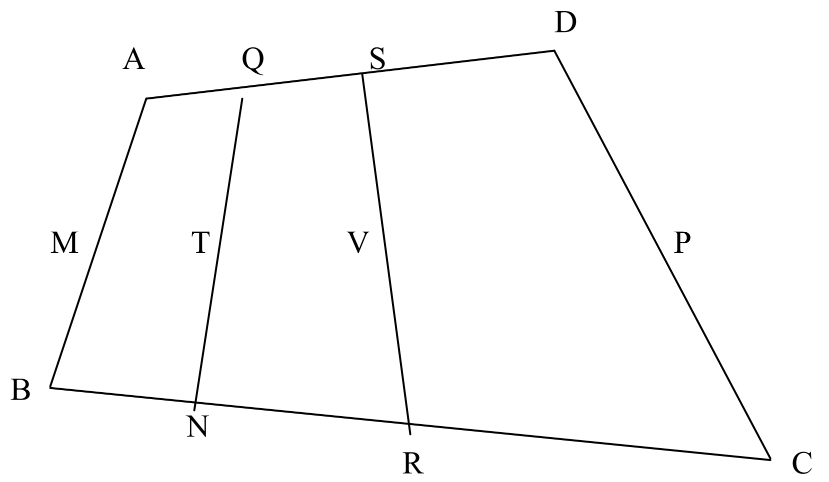
Dar $A_{ABM} = \frac{1}{2} A_{ABC}$, deci $A_{ABQ} = \frac{k}{2} A_{ABC}$ 1p

$A_{AQTB} = A_{ABQ} + A_{BQT} = \left(\frac{k}{2} + \frac{k^2}{4}\right) A_{ABC}$ 1p

Finalizare $\frac{k}{2} + \frac{k^2}{4} = \frac{5}{16} \Rightarrow k = \frac{1}{2} \Rightarrow BQ = QM$ 1p

SUBIECTUL IV

Figura ……………………………………………………………………………………1p



$S \in [AD], R \in [BC], AQ = QS = SD, BN = NR = RC, V \in [SR], SV = VR$ ……………………..1p
$MQVN$ parallelogram $\Rightarrow M, T, V$ coliniare …………………………………………………..2p
$TSPR$ parallelogram $\Rightarrow T, V, P$ coliniare …………………………………….……………..2p
Finalizare M, T, P coliniare …………………………….…………………………………..1p